"U" Substitution (integration)

The main purpose of this type of "U" substitution is to make it simpler to integrate composite functions. It basically reverses the chain rule for derivatives.

Steps:

1. Determine "u" (Often, this is the most "inner" part of the function)
2. Find the derivative of u
3. Integrate function with respect to u
4. Don't forget to add C
5. Replace u with the x form

Note: Make sure all the x's are replaced with a form of u before integrating.

Examples:

1.) $\int (x^2 - 1)^3 (2x)\, dx$ *Given function*

$u = x^2 - 1$ *Determine u and its derivative*

$\frac{du}{dx} = 2x$

$du = 2x\, dx$

$= \int u^3\, du$ *Integrate with respect to u*

$= \frac{1}{4} u^4 + C$

$= \frac{1}{4} (x^2 - 1)^4 + C$ *Replace u with the x form (Recall that $u = x^2 - 1$)*

2.) $\int (x^2 + 3x + 5)^8 (2x+3)\, dx$

$u = x^2 + 3x + 5$

$\frac{du}{dx} = 2x+3$

du = (2x+3) dx

$= \int u^8 \, du$

$= \frac{1}{9} u^9 + C$

$= \frac{1}{9}(x^2 + 3x + 5)^9 + C$

3.) $\int (1 - 2x^2)^4 (4x) \, dx$

u = 1- $2x^2$

$\frac{du}{dx}$ = -4x

-du = 4x dx

$= \int - u^4 \, du$

$= - \frac{1}{5} \, u^5 + C$

$= - \frac{1}{5} \, (1 - 2x^2)^5 + C$

4.) $\int x (4x - 5)^3 \, dx$

u = 4x-5

$\frac{du}{dx}$ = 4

du = 4 dx

$\frac{1}{4}$ du = dx

Note: If u = 4x-5, then you can solve for x to find x = $\frac{u+5}{4}$, or $\frac{1}{4}$ (u+5)

$$=\int \tfrac{1}{4}(u+5)(u^3)(\tfrac{1}{4}\,du)$$

$$=\int \tfrac{1}{16}(u^4+5u^3)\,du$$

$$=\int (\tfrac{1}{16}u^4+\tfrac{5}{16}u^3)\,du$$

$$=\tfrac{1}{80}u^5+\tfrac{5}{64}u^4+C$$
$$=\tfrac{1}{80}(4x-5)^5+\tfrac{5}{64}(4x-5)^4+C$$

"U" Substitution with Limits: (There are 2 common methods to this)

1.) $\int_1^2 (4x-5)^3\,dx$

u = 4x-5

du = 4dx

$\frac{1}{4}$ du = dx

$= \int_{-1}^{3} \frac{1}{4}u^3\,du$ (Note the change in the limits)

x = 2 → u = 4(2)-5 = 3

x = 1 → u = 4(1)-5 = -1

$= \frac{1}{16}(3)^4 - \frac{1}{16}(-1)^4$

$= 5$

OR

2.) $\int_1^2 (4x-5)^3 \, dx$

u = 4x-5

du = 4dx

$\frac{1}{4}$ du = dx

$= \int_1^2 \frac{1}{4}u^3 \, du$

$= \left[\frac{1}{16}(4x-5)^4\right]$

$= \frac{1}{16}(4(2)-5)^4 - \frac{1}{16}(4(1)-5)^4$

$= 5$